

Quiz 2

Be sure to show work clearly.

(1) Given the points P(3, -2, 0) and Q(1, 4, 7)

(4 points)

a) Find the distance between P and Q.

$$\sqrt{89}$$

b) Find the vector \overrightarrow{PQ}

$$\langle -2, 6, 7 \rangle$$

(2) Given the vectors $\mathbf{a} = \langle -5, 1, 2 \rangle$ and $\mathbf{b} = \langle -1, 0, 4 \rangle$, find

a) the angle between \mathbf{a} and \mathbf{b}

(2 points)

(Note: In this class, exact answers should always be given unless otherwise stated; that is don't use your calculator to get an approximation)

$$\cos \theta = \frac{\mathbf{a} \cdot \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{13}{\sqrt{30} \sqrt{17}}$$

$$\theta = \cos^{-1} \left(\frac{13}{\sqrt{510}} \right)$$

b) $\mathbf{a} \times \mathbf{b}$

(10 points)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -5 & 1 & 2 \\ -1 & 0 & 4 \end{vmatrix} = \langle 4, 18, 1 \rangle$$

Check your answer by showing it is orthogonal to both \mathbf{a} and \mathbf{b} .

$$\langle 4, 18, 1 \rangle \cdot \langle -5, 1, 2 \rangle = 0$$

$$\langle 4, 18, 1 \rangle \cdot \langle -1, 0, 4 \rangle = 0$$

Note:
 $\mathbf{a} \times \mathbf{b}$ is always orthogonal to both \mathbf{a} and \mathbf{b} , so these should be zero. I suggest doing this on all your cross products.

c) $\text{proj}_{\mathbf{a}} \mathbf{b}$

(2 points)

$$\text{proj}_{\mathbf{a}} \mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{\mathbf{a} \cdot \mathbf{a}} \mathbf{a}$$

$$\left\langle -\frac{13}{6}, \frac{13}{30}, \frac{13}{15} \right\rangle$$

projection on \mathbf{a} is a multiple of \mathbf{a}

d) a unit vector in the direction of \mathbf{b}

(2 points)

$$\hat{\mathbf{u}} = \frac{1}{\|\mathbf{b}\|} \mathbf{b}$$

$$\left\langle -\frac{1}{\sqrt{17}}, 0, \frac{4}{\sqrt{17}} \right\rangle$$